Supersingular Isogeny Key Encapsulation (SIKE)

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Michael Burkhalter, Lauro Böni

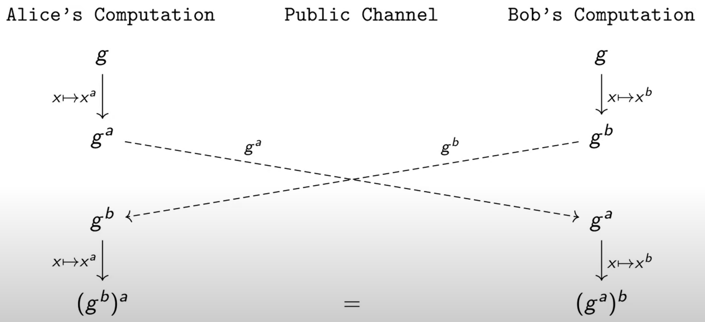
Krypt Det

Bern, 3. September 2020

Overview

Recap: Classical Diffie-Hellman (DH) protocol

Setup: Fix a group G and $g \in G$.



Hard problem: Given g, g^a, and g^b, determine g^{ab}.

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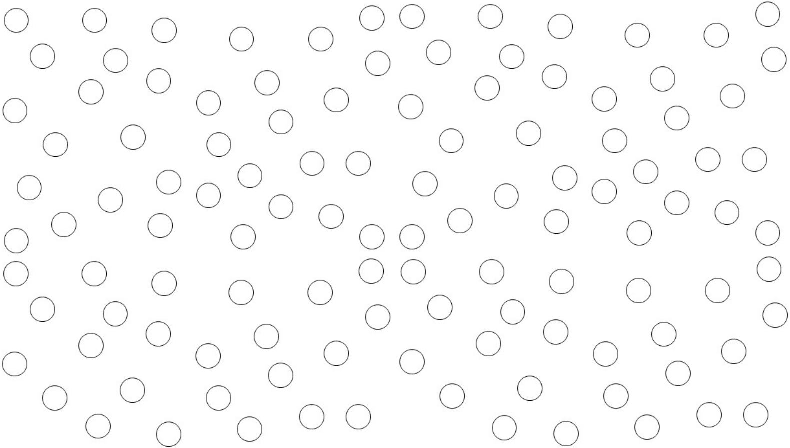

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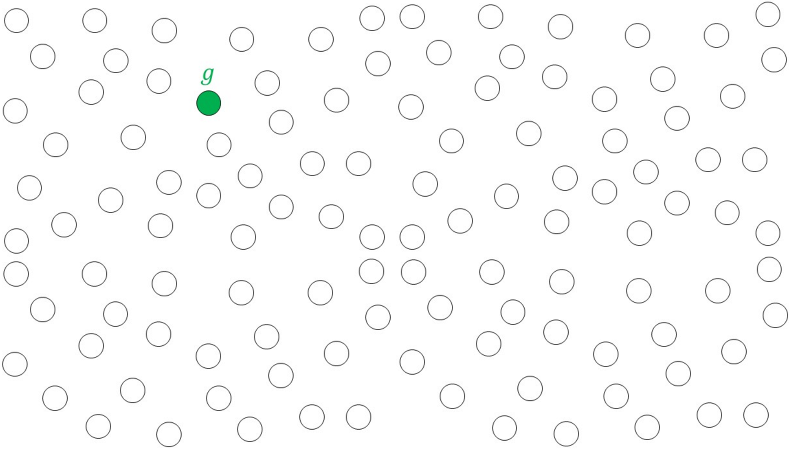

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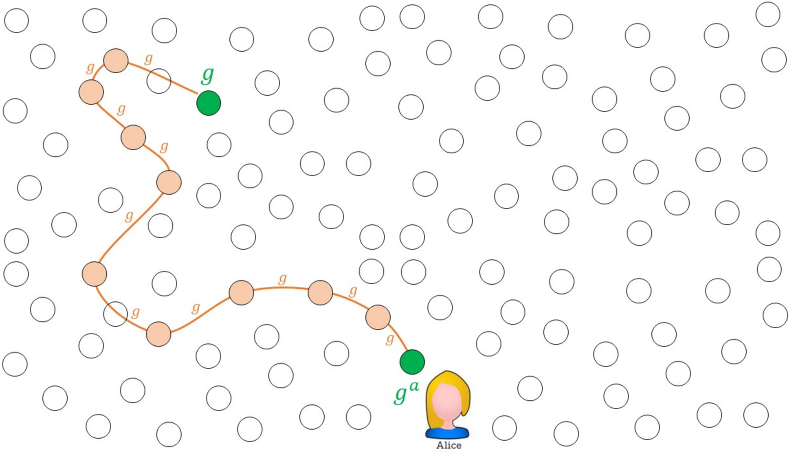

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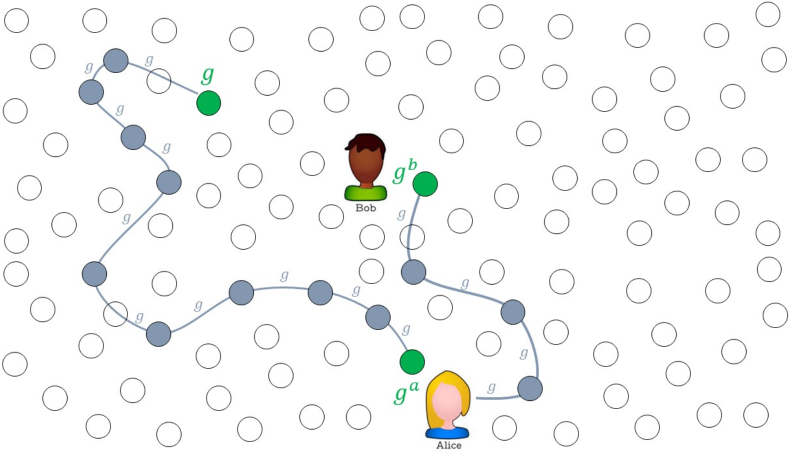

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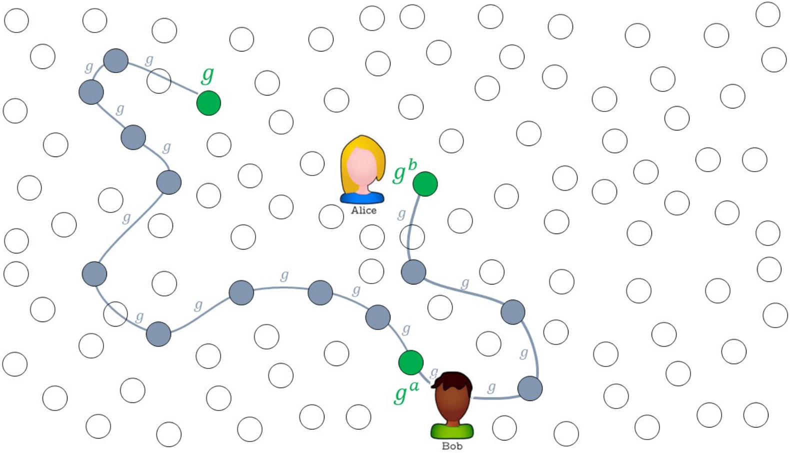

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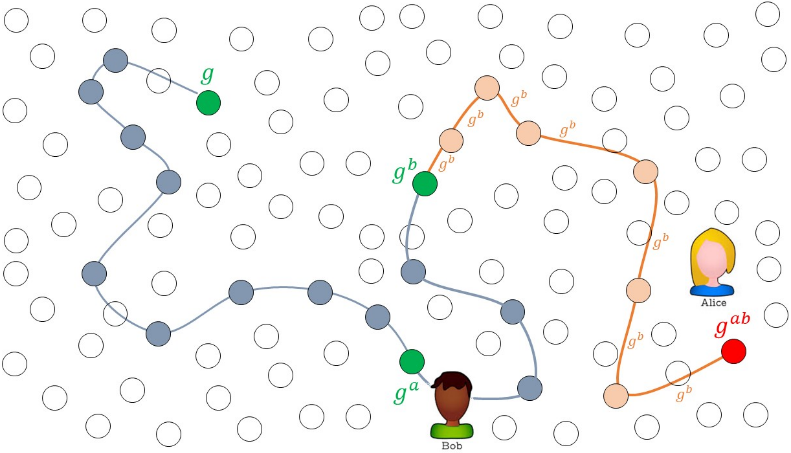

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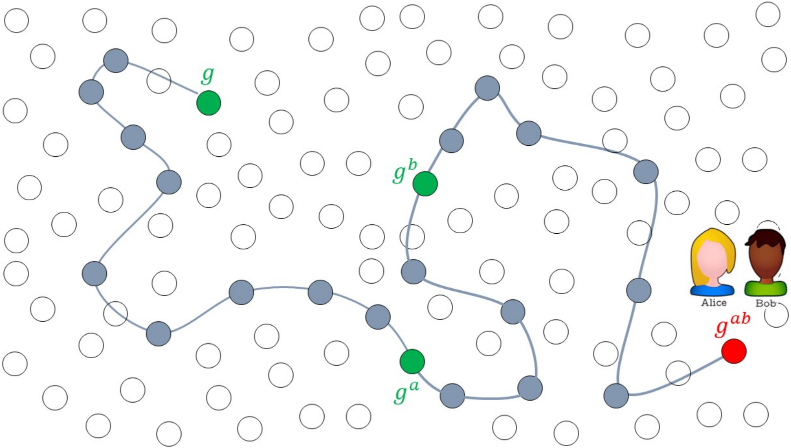

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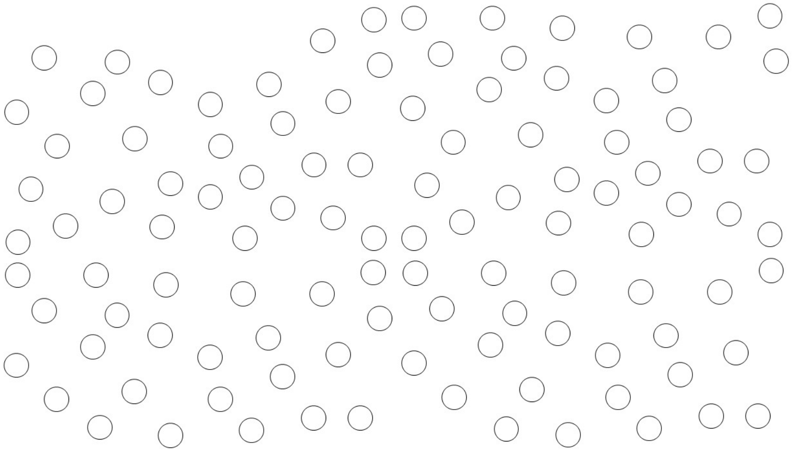

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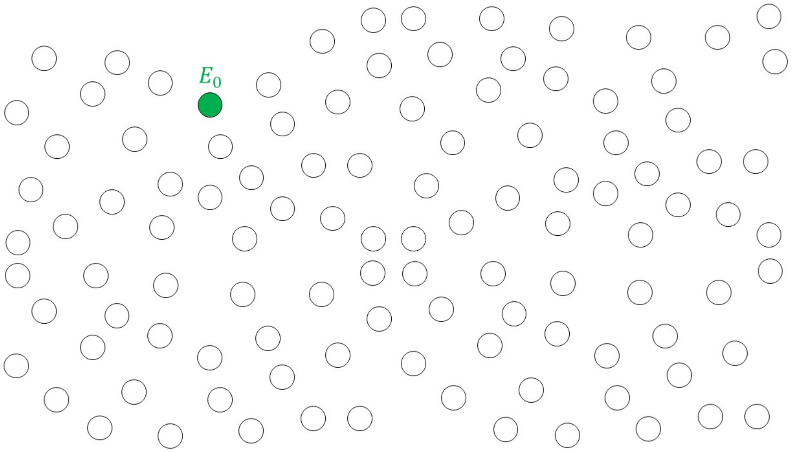

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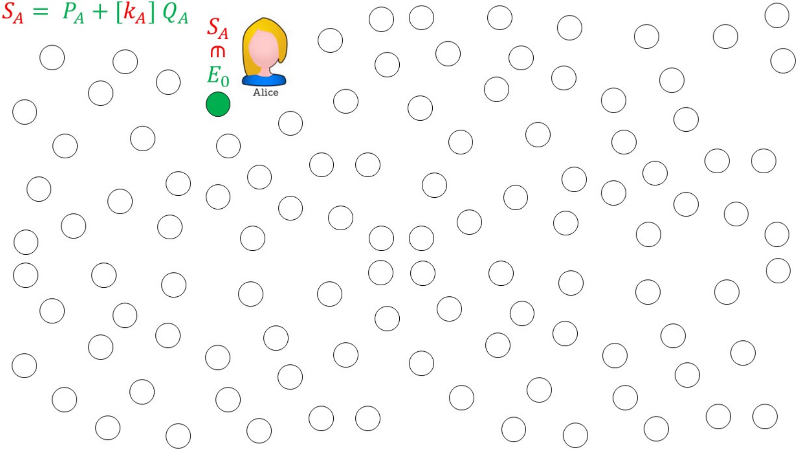

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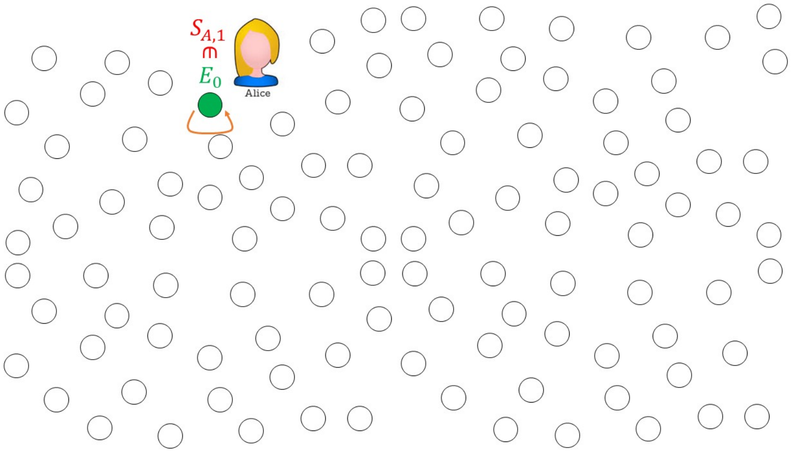

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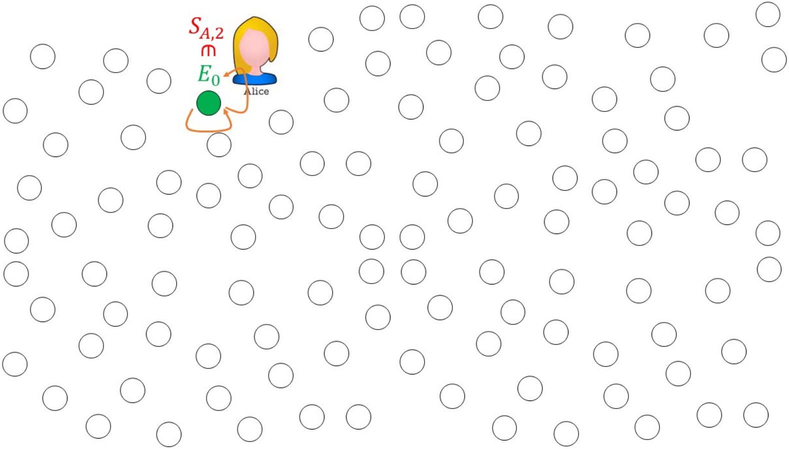

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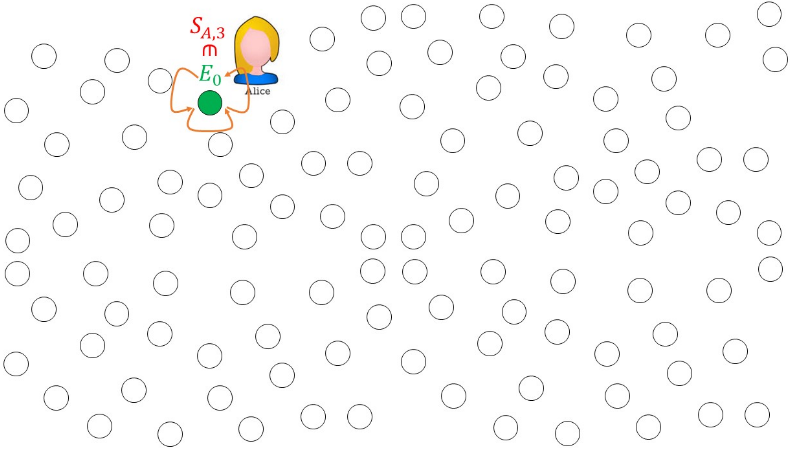

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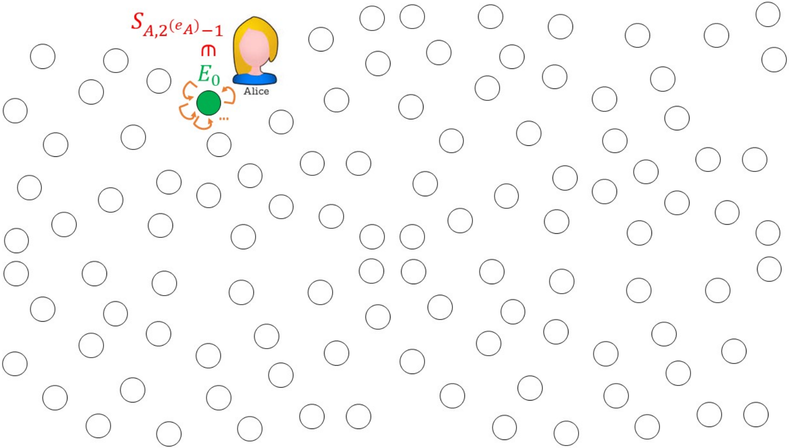

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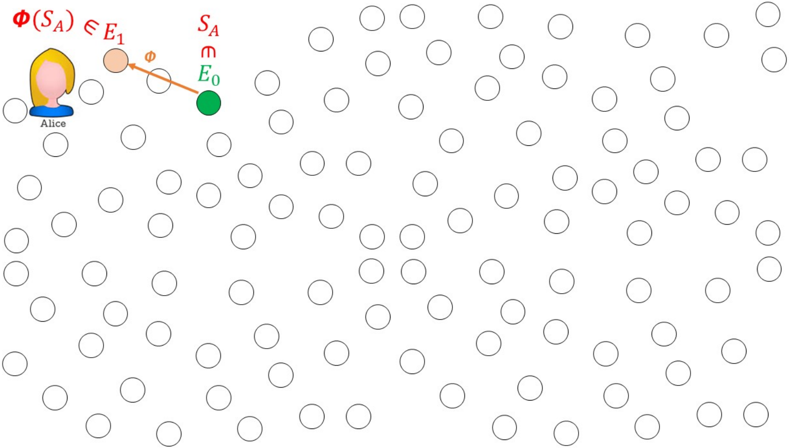

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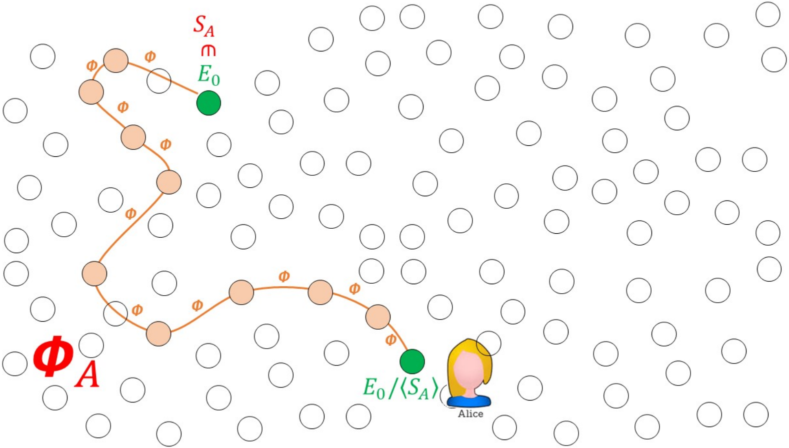

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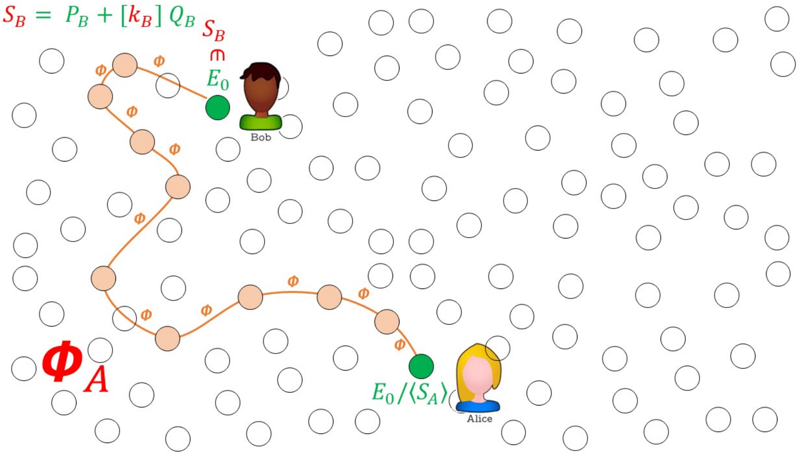

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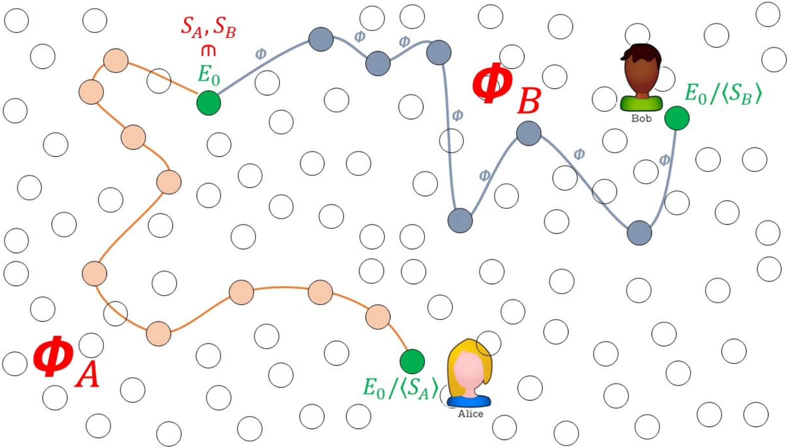

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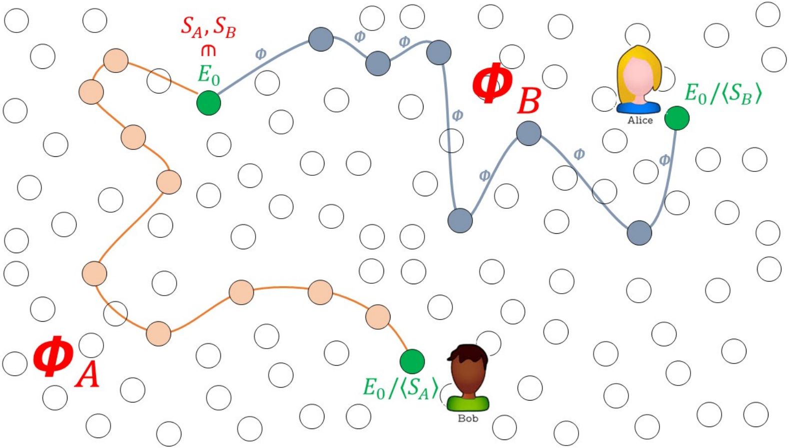

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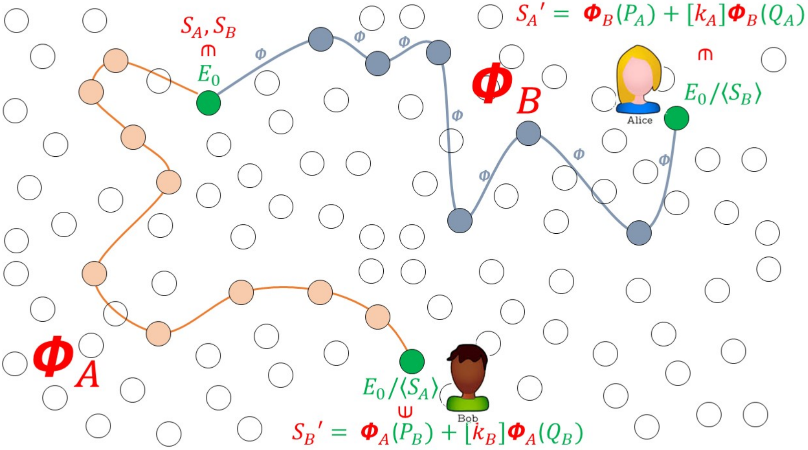

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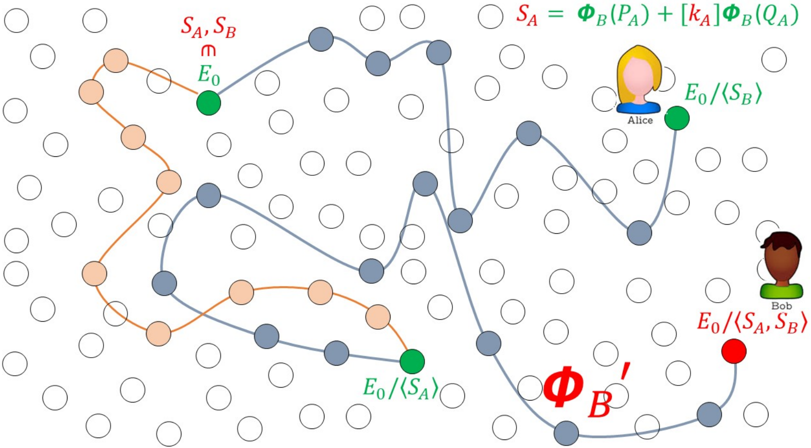

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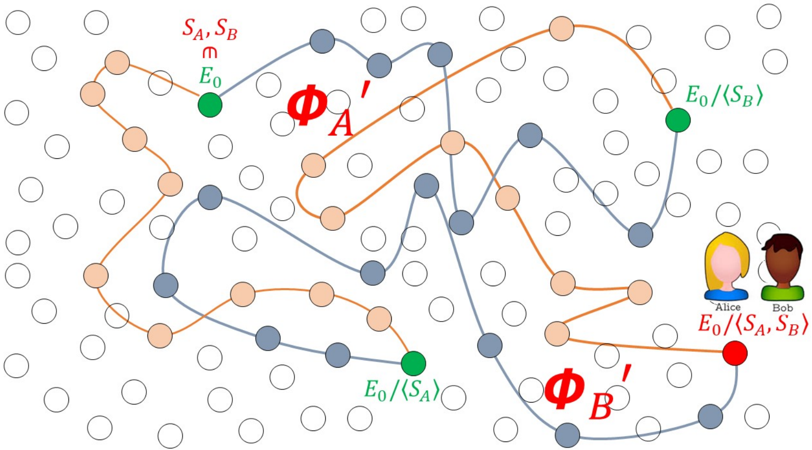
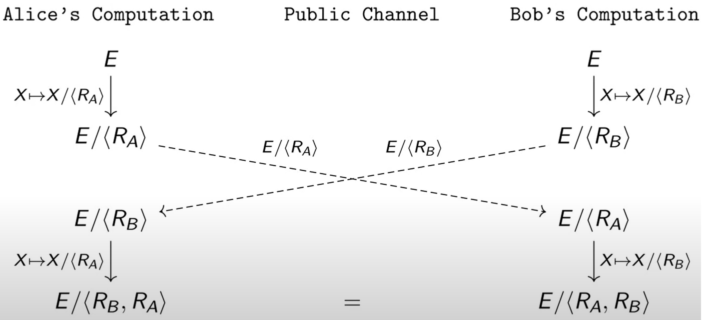


Supersingular Isogeny Diffie-Hellman (SIDH) protocol

Setup: Fix a supersingular isogeny class $\mathcal{C}$ and $E \in \mathcal{C}$.



Hard problem: Given E, $E/\langle R_A \rangle$, $E/\langle R_B \rangle$ *, determine $E/\langle R_A, R_B \rangle$.
* Some extra information is also available.

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Practical implementation

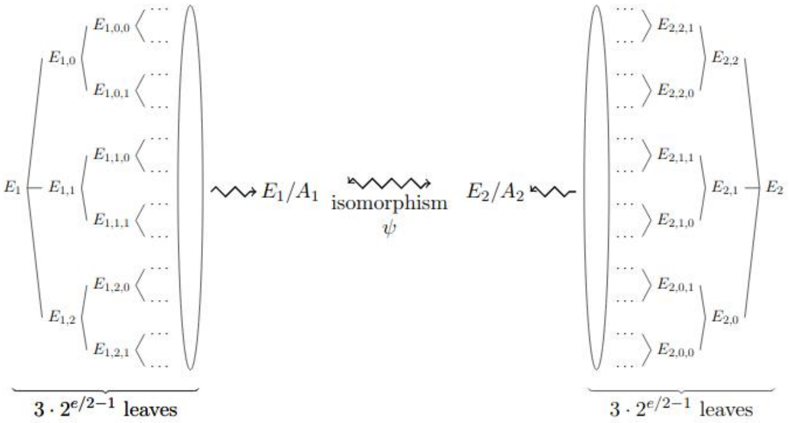
Meet-In-The-Middle

Underlying Math Problem:

Given public parameters $l_A, l_B, e_A, e_B, p, E, P_A, Q_A$ and $E/\langle S_A \rangle$: Compute the $l_A^{e_A}$-isogeny $E \to E/\langle S_A \rangle$

- e_A steps in the l_A-isogeny graph are much fewer than the average number of steps necessary to join any two nodes
- Very likely that the e_A steps represent the shortest path between E and $E/\langle S_A \rangle$
- Build list of all destination nodes taking $e_A/2$ steps from E
- For each destination of length-$e_A/2$ walks from $E/\langle S_A \rangle$, compare to list until match is found

Schematic Of Meet-In-The-Middle Attack



Costs of Classical Attacks

- Classical run time $\mathcal{O}(p^{1/4})$
- $\mathcal{O}(p^{1/4})$ memory needed to build all walks from E
- Smallest SIKE prime has 434 bits makes memory needs prohibitively large
- Technical enhancements give slower algorithms when memory is limited (e.g. to $\sim 2^{80}$)

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EXPONENTIAL IN TIME AND SPACE

PQC security definition

NIST security strength categories

Computational resources required to break security definition

$\geq$

resources for key/collision search on AES/SHA3

NIST level	classical gates	reference algorithms	factoring	discrete logarithm key	discrete logarithm group	Elliptic curve	SIKE
1	2^{143}	AES-128	3 072	256	3 072	256	SIKEp434
3	2^{207}	AES-192	7 680	384	7 680	384	SIKEp610
5	2^{272}	AES-256	15 360	512	15 360	512	SIKEp751

Quantum complexity is . . .

- expressed in terms of classical gates
- based on NIST's restriction on a maximal running time of a quantum circuit

Performance & resources

Comparison

classical Elliptic Curve with 256-bit prime $\iff$ `SIKEp434`
(both corresponding to security level 1, `AES-128`)

	prime bits	secret key bytes	public key bytes	shared secret bytes	cycles
EC	256	32	64	64	$\sim 4\,000\,000$
SIKE	434	330	374	16	$\sim 25\,000\,000$

executed on a 2.7 GHz Intel Core i5-5350U (Broadwell) processor

other resources for SIKE protocol

- between $O(10^7)$ and $O(10^8)$ cycles
- timings of $O(1)$ ms
- 70-80 mW energy consumption (on efficient ARM M4-Cortex processor)

The race for a new quantum-safe standard

What position does SIKE take?

small key sizes

- 564B public keys/48B private keys (for security level 5) compared to kB/MB range for other quantum-safe protocols

increased runtime by a factor of around 100

- seconds instead of miliseconds

Reason for SIKE to still be in the race

- EC theory well-proven in crytographic theory
- quantum attack algorithms not yet investigated enough
- desire for broad range of hardness assumption